Superbosonization

(a new effective-field method for random matrix and disordered electron systems with local gauge symmetries)

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Symmetric spaces

Riemannian symmetric superspaces: definition

Consider sections of (F_x) M (superfunctions),

M with values (x) (F_x)

Example

$$_{1} \quad U \quad /(U \quad U)$$

Example (continued)

V_0^{p}	0	0	1	1
V_0^{q}	0	0	1	1
V_1^{r}	1	1	0	0
V_1^{s}	1	1	0	0

Lie superalgebra $(\ {}_0 \quad {}_0)\ (\ {}_1 \quad {}_1)$

$$(\ ^{p\ q} \,|\, ^{r\ s}) \qquad {}_{p\ \ q|r\ \ s}$$

Important:

$$\left\langle \mathrm{e}^{\mathrm{i}\,\mathrm{Tr}\,HK} \right\rangle_{\mathrm{GOE}} \qquad \mathrm{e}^{\mathrm{i}\,\mathrm{Tr}\,HK} \qquad (\quad) \qquad \mathrm{e}^{\;(b^2/2N)\,\mathrm{Tr}\,K^2}$$

Background & motivation (continued)

Pruisken-Schäfer domain

$$Rv \quad v : \quad \begin{array}{ll} v^t s v \quad 0 & \text{space-like,} \\ v^t s v \quad 0 & \text{time-like,} \quad \mathbf{0} \end{array}$$

Every $O_{p,q}$ diagonalizable matrix R has p space like and q time like eigenvalues.

Encode ordering by motif, e.g., $(R) \quad \mathbf{0} \ \mathbf{0}\mathbf{0} \quad (p \quad q \quad 3)$.
Associate with each motif a domain D by closure.

Pruisken Schäfer domain $D \quad \bigcup \ D$ is a union

of $\begin{matrix} p & q \\ & p \end{matrix} \quad \begin{matrix} p & q \\ & q \end{matrix}$ domains. Each D is $O_{p,q}$ invariant.

$D \quad D$ for has co dimension 2.

Statement of result

Let $|dR| \qquad {}_{i \quad j} dR_{ij}$ (Lebesgue measure)

Theorem (FWZ). There exists some choice of cutoff function

$$(\quad)\;(\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0)$$

$$\operatorname{sgn}(\quad) \qquad 1 \qquad\qquad\qquad\qquad C_{\;,}$$

Corollary

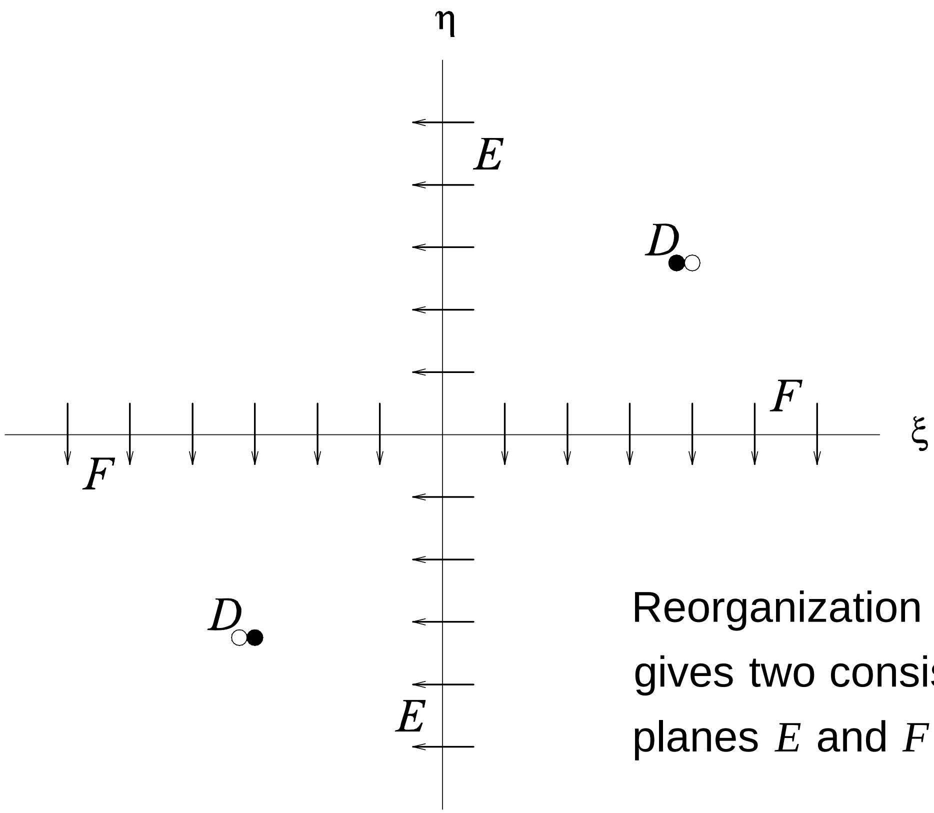
Formulation in terms of eigenvalues: Let $R \quad g \quad g^{1}$
with $\mathrm{diag}(\ _1,\ldots,\ _{p\ q})$ and $g \quad SO_{p,q}$. Volume element
$|dR| \quad J(\quad)|d \quad|dg$ where $|d \quad| \quad \prod_{i\ 1}^{p\ q} d \ _i$ and dg Haar
$SO_{p,q}$. J cs 0 0.35294 0.78431 scn 184.26 73.431 10.38 11.22 ref $J(\quad) \quad |\ _i \quad _j|$

Two domains for p = q = 1

R $\quad r_{11} \quad r_{12}$

Light-cone coordinates:

Reorganization of boundary components



Reorganization of the boundary pieces gives two consistently oriented boundary planes E and F.

Superbosonization

- P. Littelmann, H.-J. Sommers, M.R.Z., Commun. Math. Phys. (in press)
- J.E. Bunder, K.B. Efetov, V.E. Kravtsov, O.M. Yevtushenko, M.R.Z., J. Stat. Phys. 129 (2007) 809

Motivation: supersymmetry method

Central object of the theory is the characteristic function of the

on, (K) $\mathrm{e}^{\mathrm{i}\,\mathrm{Tr}\,HK} d$ (H), where $(Z$ commuting,

anticommuting variables) K p Z $\tilde{Z}$ q $\sim$

Special case: commuting variables only

Let $p \quad 1, q \quad 0$ and consider GL_N invariant holomorphic function $f : {}^N \quad ({}^N) \qquad , \; f(z,\tilde{z}) \quad f(gz,\tilde{z}g^{\;1}), \; g \quad \mathrm{GL}_N$.

Fact (from invariant theory): there exists a holomorphic function $F : \qquad$ such that $F(\tilde{z} \;\; z) \quad f(z,\tilde{z})$.

By push forward of the integral one has

$${}_N \, f(z,z\;\;)\, d^{2N} z \quad c_N \quad F(r)\, r^{N \;\; 1} dr$$ (if the integral exists).

generalization: see Fyodorov, Nucl. Phys. B **621** (2002) 643

Special case: Grassmann variables only

Let $F:$ be a holomorphic function. For a vector

$_1, \ldots,$ $($ $)$.

$($ $)$:

Symmetry argument (heuristic)

Pullback $f \quad F$. Compare two distributions:

Distribution 1: $_1[F] \quad {}_{\tilde{Z} \;\; Z} \quad F$

Distribution 2: $_2[F] \quad {}_M DQ \; \mathrm{SDet}^{N/2}(Q) \, F(Q)$

For $g \quad \mathrm{OSp}_{2p|2q}$ let $F_g(Q) : \quad F(gQg^T)$.

The transformation behavior is the same:

$$_A[F_g] \quad \mathrm{SDet}^{\,N}(g) \quad {}_A[F] \qquad (A \quad 1, 2).$$

Comments

Application: Wegner's N-orbital model, U(N)

Hilbert space $V \quad {}_i \quad V_i$, orthogonal projector $\quad {}_i : V \quad V_i$

$$: \quad e^{\mathrm{i}\,\mathrm{Tr}\, HK} d \quad (H) \quad e^{(1/2N) \quad {}_{i,j} C_{ij} \,\mathrm{Tr}\, K \quad {}_i K \quad {}_j}$$

polynomials :